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DELAWARE ESTUARY COMPREHENSIVE STUDY. TECHNICAL REPORT NUMBER 3. MINIMIZING COSTS OF INVESTMENTS TO IMPROVE WATER QUALITY: AN ILLUSTRATIVE PROBLEM

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Delaware Estuary Comprehensive Study

Report Number 3

Division of Water Supply and Pollution Control
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U.S. DEPARTMENT OF HEALTH, EDUCATION, AND WELFARE
Public Health Service

TECHNICAL REPORT NO. 3

Minimizing Costs of Investments to Improve Water Quality: An Illustrative Problem

Water quality control is similar in many ways to other managerial enterprises in our society. In particular, costs as well as benefits are associated with every program for water quality management. If such programs are to be planned rationally, costs and benefits should be analyzed whenever possible. This should preclude, for example, adoption of a program if there exists another program whose benefits are identical but whose costs are less.

When an environment is complicated, however, it is often difficult to determine what constitutes an optimal managerial policy. The Delaware Estuary is an example of such an environment. It is studded with tributaries, waste discharges, reaches whose volumes can be changed by dredging, other factors which affect water quality and other alternatives to control water quality. All of these can be combined in a vast number of ways. In fact, any particular of goals for water quality improvement usually can be attained by any one of a very large number of combinations of alternative actions.

Consider, for example, the number of alternative combinations which exist in the following situation which is less complicated than the Delaware Estuary. Assume that there are twenty-five waste discharges, each of which can be modified to five successive extents. There are, therefore, 5^{25} different combinations of modifications. If a computer would take only one microsecond to evaluate the cost and effectiveness of each combination, it would take approximately one hundred centuries to compare all the possible combinations. It is imperative, therefore,

that in the planning of managerial strategy, more powerful analytical techniques be drawn upon than an enumeration of all alternatives.

In general, those combinations of alternatives whose effects will be at least as favorable as specified by the goals are called "feasible" combinations. Associated with each feasible combination is its cost of accomplishment. A feasible combination is "optimal" for a set of goals if no other feasible combination will attain the goals at lower cost.

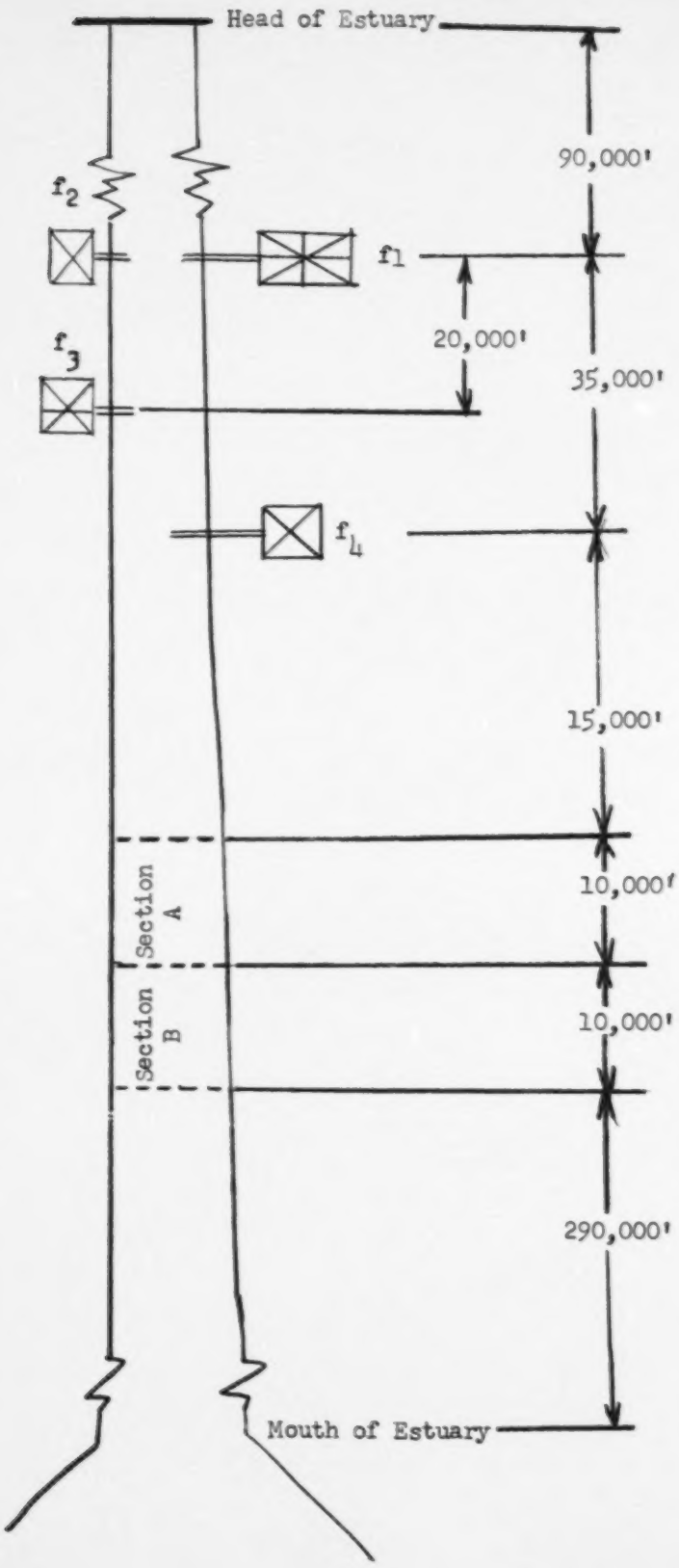
Before promulgating goals for water quality improvement, it would be useful to know the optimal policies (and the minimal cost) for each of a variety of goals in addition to the goals that are chosen. Such an analysis would promote the practice of carefully considering the relative effects of attaining goals before their desireability is publicly proclaimed.

This is possible only if the computational effort to examine the alternative combinations is not prohibitively lengthy and, therefore, too expensive. Fortunately, there exist expedient computational techniques to determine optimal solutions to many significant water quality management problems. These techniques have been developed by synthesizing results from the applied sciences of communications engineering and operations research. In the next section, the usefulness of these methods will be illustrated with a water quality management problem. Although the situation is hypothetical, its salient characteristics are similar to some problems in the Delaware Estuary.

Illustrative Investment Problem

Assume that the water quality of an hypothetical estuary is to be improved. In particular, it is agreed that the dissolved oxygen

FIGURE 1: Layout for Illustrative Problem



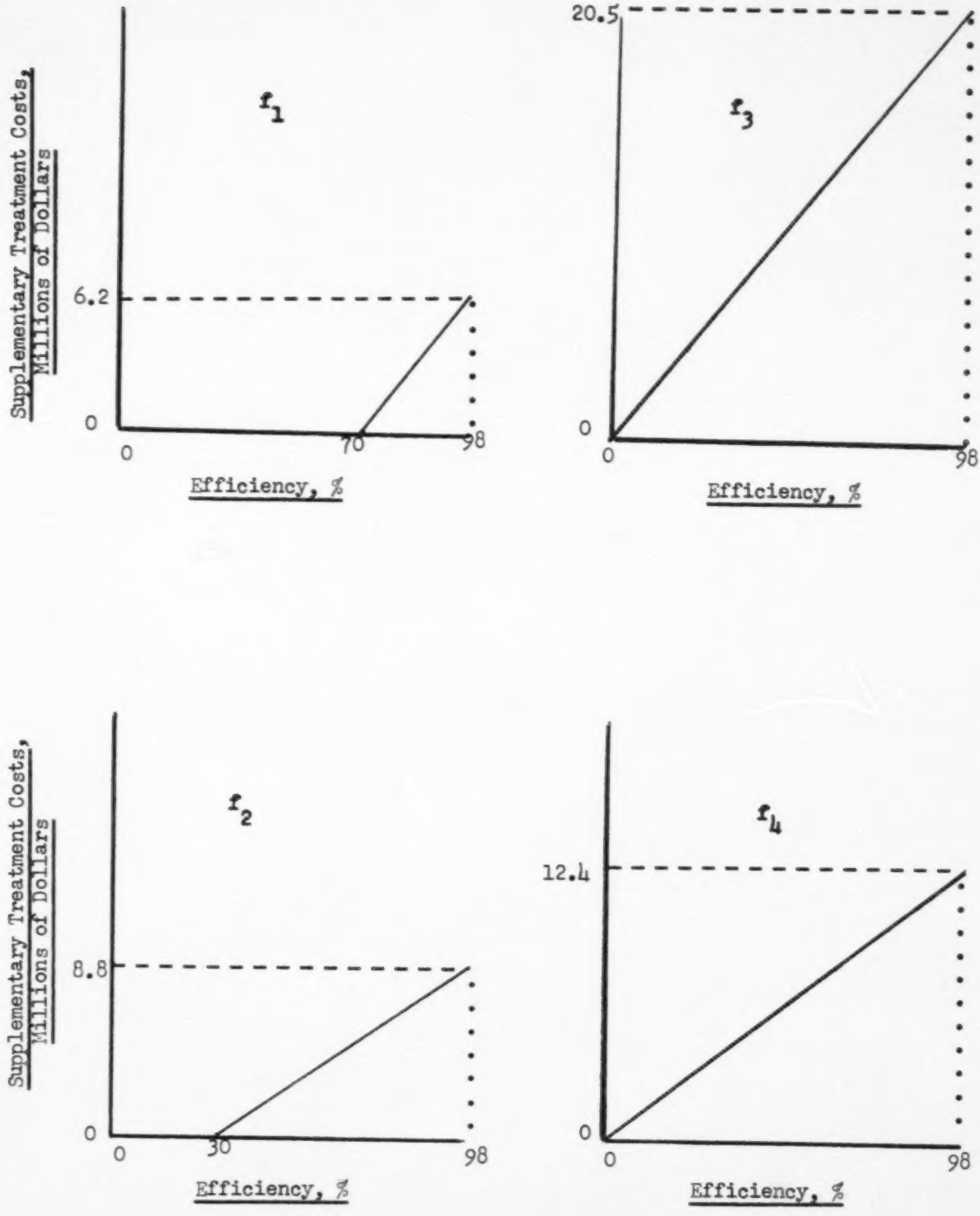
f_1: Effluent from municipal STP with 100 mgd influent at 350 mg/l ultimate BOD of which 70% is removed.

f_2: Effluent from industrial STP with 50 mgd influent at 700 mg/l ultimate BOD of which 30% is removed.

f_3: Untreated municipal wastes at a rate of 100 mgd at 350 mg/l ultimate BOD.

f_4: Untreated industrial wastes at a rate of 50 mgd at 700 mg/l ultimate BOD.

FIGURE 2: Hypothetical Costs of Supplementary Waste Treatment



concentrations (D.O.) during the summer and autumn are too low in two adjacent sections of the estuary, sections A and B. Assume that only four alternatives are available to increase the D.O.: there are four effluents, entering the estuary at various places, each of whose waste loads may be reduced by modifying the present treatment process. The hypothetical situation is pictured in Figure 1.

Assume that the goals under consideration are to increase the mean D.O. (during summer and autumn) in section A from two to three mg./l. and, in section B, to increase it from one to three mg./l. This would require increasing the D.O. by one mg./l. in A, and by two mg./l. in B. It would also be of interest to know how much more expense would be necessary if the goal in A were to be to increase the mean from two to at least four mg./l. (an increase of two mg./l.) instead of only to three mg./l. (an increase of one mg./l.).

This problem has been analyzed so as to minimize the net cost of the new facilities whose construction would be necessary to raise the D.O. The data that were used were of two kinds:

(1) Hydrodynamic data that are representative of the Delaware Estuary;

(2) Hypothetical costs of constructing facilities to supplement or increase the efficiency of present waste treatment plants (Figure 2).

An optimal solution* to increase the D.O. in A by at least 1 mg./l. and in section B at least 2 mg./l. is:

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*The solution was obtained from a linear programming formulation of the problem. See the Appendix to this paper for an exposition of this method of solution.

I $\begin{cases} f_1: & \text{Make no improvement; leave at present efficiency of 70\%;} \\ f_2: & \text{Increase the efficiency from 30\% to 98\%;} \\ f_3: & \text{Make no improvement; presently has no treatment.} \\ f_4: & \text{Construct a plant with 30\% efficiency.} \end{cases}$

The minimal cost (generated only by I, the solution above) is $12,660,000. In other words, no other solutions exist which would attain the goals at A and B at a cost lower than $12,660,000.

Suppose that solution I is considered politically unacceptable. Note that f_2 is going to increase its treatment from 30% to 98% while f_3 is to be permitted to continue with no treatment at all. A regulatory body might, for example, consider imposing the following restriction:

R: All effluents must have undergone at least primary treatment causing a reduction of at least 30% in ultimate B.O.D.

Because of f_3, solution I is inadmissable under restriction R.

Before legislating restriction R, however, a regulatory body ought to be aware of R's effect on the cost of an optimal solution to the investment problem. An optimal solution* that will increase the D.O. in A by at least 1 mg./l., in B by at least 2 mg./l. and construct plants of at least 30% efficiency at f_3 and f_4 (the consequences of R) is:

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*This solution was obtained from a linear programming formulation of the problem. See the Appendix to this paper for an exposition of this method of solution.

I_R {
- f_1: Make no improvement; leave at present efficiency of 70%;
- f_2: Increase the efficiency from 30% to 73%;
- f_3: Construct a plant with 30% efficiency;
- f_4: Construct a plant with 30% efficiency.

The minimal cost (generated only by I_R) is \$15,410,000. This is \$2,750,000 more than the cost of I. This difference in costs is the premium that would have to be paid to attain the quality improvement goals if restriction R were to be imposed.

Both A and B are downstream from the points at which the effluents enter the estuary. Therefore, any modifications which benefit the quality in A also imprive it in B (but, in this hypothetical situation, by a greater amount than in A). As a result, in order to increase the D.O. in B by 2 mg./l., both solutions I and I_R may have had to simultaneously increase the D.O. in A by more than 1 mg./l. Analysis of the solutions showed this to be true:

(1) Solution I increased B by 2 mg./l. and increased A by 1.87 mg./l.

(2) Solution I_R increased B by 2 mg./l. and increased A by 1.86 mg./l.

Because solutions I and I_R are optimal for their respective problems, no sacrifices can be made in the improvements in A that will succeed in reducing the minimal costs (without also reducing the improvement in B). If a regulatory agency were to face this problem,

it might consider raising the D.O. in A by 2 mg./l. because it must, in any event, raise it by 1.86 mg./l. An optimal solution* to the problem of raising the D.O. in both A and B by at least 2 mg./l. is:

II
- f_1: Make no improvement upon the present 70% efficiency;
- f_2: Increase the efficiency from 30% to 98%;
- f_3: Permit the effluent to remain untreated;
- f_4: Construct a plant to operate at 40% efficiency.

The cost of II is $13,880,000, an amount $1,220,000 higher than the cost of I. This difference is the added cost to raise the D.O. at A all the way up 2 mg./l.

Note that II is similar to I: f_3 is permitted again to issue untreated. What, one may wonder, would be the incremental cost of superimposing restriction R (at least 30% efficiency) to the problem whose optimal solution was II? The optimal solution* to the problem of increasing the D.O. in both A and B by at least 2 mg./l. while simultaneously abiding by R is:

II_R
- f_1: Make no improvement; leave at present efficiency of 70%;
- f_2: Increase the efficiency from 30% to 80%;
- f_3: Construct a plant with 30% efficiency;
- f_4: Construct a plant with 30% efficiency.

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*This solution was obtained from a linear programming formulation of the problem. See the Appendix to this paper for an exposition of this method of solution.

The cost of II_R is \$16,310,000. This cost is \$2,430,000 higher than that of II. This difference is the premium that would have to be paid were restriction R to be included with the attainment of two mg./l. (of D.O.) improvements in both A and B.

The costs of the four optimal solutions can be summarized:

	Increase A at least 1 mg./l. Increase B at least 2 mg./l.	Increase A at least 2 mg./l. Increase B at least 2 mg./l.
No Additional Restriction	I: \$12,660,000	II: \$13,880,000
Under Restriction R (At least 30% treatment)	I_R: \$15,410,000	II_R: \$16,310,000

Note that the cost increases as the water quality goals become more stringent or as side restrictions (e.g. restriction R) are added.

Allocation of Costs

In this section, the implications of solutions I, II, I_R and II_R and the motivation for restriction R will be examined more closely.

The four problems in the previous section were posed so as to minimize a solution's cost to all sectors of the estuarial region's economy. Thus, if it was more economical to improve existing plants than to construct them where none existed, this was indicated in the optimal solution. Accordingly, restriction R was considered. Unfortunately, this caused costs to increase.

The imposition of restriction R is not, however, the only method with which the allocation of efforts can be made (what may be considered by some to be) more equitable. An optimal solution's cost need not be

borne by the same institutions at which investment funds are to be spent. In solution I, for example, improvements at f_2 need not be paid for by the municipality at f_2. The company that owns f_3 might, at a cost lower than instituting 30% removal, be willing to pay for some of the improvements at f_2. This is, in effect, the choice that is made whenever a company is situated adjacent to the Estuary but chooses to discharge its wastes into a municipal system rather than build its own.

In summary, a solution which is optimal according to economic efficiency specifies how each alternative site should be modified. The equitable allocation of the costs of the optimal pattern of modifications, however, is entirely another matter and beyond the scope of this paper.

Concluding Remarks:

The preceding example has attempted to illustrate the flexibility that exists in the formulation of managerial problems in water quality control. The optimal solutions to such problems will indicate the consequences of promulgating various quality improvement goals and imposing alternative socio-political constraints. The allocation of the resultant minimal costs, however, can be accomplished in numerous ways. The choice among these is beyond the scope of this paper.

APPENDIX: Linear Programming Solution to the Investment Problem*

Consider the following problem:[1]

At several places in the estuary it is desired to increase the equilibrium quality (e.g. mean DO) by at least specified minimum amounts. For this purpose, funds will be invested in the improvement or creation of various facilities along the estuary. The manager wishes to know the minimum total investment that it is necessary to make and how this total should be allocated among the alternative sites and facilities.

This problem can be posed as a linear programming problem, a mathematical model, for whose solution there are well-known computational algorithms.[2] The data that are necessary for its solution are of two kinds:

(a) At each potential site or existing facility, one must know the costs associated with the range of alternative actions that can be taken there. This relationship is the marginal cost function at that site or facility.

(b) It is necessary to be able to predict how an action taken at any site or facility will affect the qualities at each of the places (in the estuary) whose improvement is sought. This information is embodied in the transfer functions which were introduced earlier.

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*Drawn from Thomann, R. V. and M. J. Sobel, "Estuarine Water Quality Management and Forecasting," presented at ASCE Water Resources Conference, Milwaukee, Wisc., May 15, 1963. pp. 25-30.

[1]Sobel, M. J., "On the Management of the Quality of Natural Water Systems," Contributed Paper M-1, Meeting of the Operations Research Society of America at Cleveland, Ohio, May 28, 1963.

[2]Gass, S. I., Linear Programming, Methods and Applications, (1958) New York: McGraw-Hill.

In a particular situation in which the mean DO is to be improved, for example, suppose a potential facility is a municipal plant with primary treatment. It would be necessary to know the costs of removing 10%, 20%, 30%, etc. of the B.O.D. which is presently in the effluent. Furthermore, the effects of these alternative improvements on the quality at various places must be foretold. These functions (the costs and the effects of alternative improvements) are needed for each facility from which it is possible to affect an improvement in the estuary.

How an estuary transforms its inputs into its state vector is inferred in equation (1).* The details of this transformation are in

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*The following definitions and relations were presented earlier in the paper from which this Appendix is drawn:

$f_i(t)$ = input, e.g. effluent, into the i-th segment of the estuary (as a function of t, time), i = 1,..., n.

$c_j(t)$ = response, to the inputs, of quality in the j-th segment of the estuary (as a function of time), j = 1, ..., n.

$(c)' = (c_1(t), c_2(t), .., c_n(t))$ = the transpose of the state vector

$(f)' = (f_1(t), f_2(t), ..., f_n(t))$ = the transpose of forcing functions (inputs).

Equation (1): $(T)(c) = (f)$

where (T) = a matrix of transfer functions.

Equation (6): $(c) = (\emptyset)(f)$

where $(\emptyset)$ = the inverse of (T).

The response in segment j to an input in segment i is given by

Equation (7): $c_j(t) = \emptyset_{ij} f_i(t)$

where $\emptyset_{ij}$ is the function in the i-th row of the j-th column of $(\emptyset)$.

(continued on next page)

equations (7), (8) and (9) from which it is clear that the state in a particular section is the sum of the appropriate transformations of inputs into all sections, i. e.

(9) $\quad c_j(t) = \sum_i f_{oi} A_{ij}(\omega)\cos(\omega t + \theta_{ij}(\omega))$.

In the following analysis, the periods of the forcing functions will be regarded as indefinitely large. This leads to treatment of $\{f_i(t)\}$ as a set of constants. This assumption is reasonable in the context of a manager's investment problem. He is planning the construction of facilities which will be in operation many years in the future. Furthermore, he is interested in improving the annual mean quality rather than fluctuations about that mean during the year (problems of transient control will be analyzed in a later section of this paper). Accordingly, as T (the period of a forcing function) gets large, $\omega = 2\pi/T$ gets small and, in the limit, goes to zero. It can be shown [p.5 of Solodovnikov, Selected References] that the phase shifts $\{\theta_{ij}(\ )\}$ are arctangent functions whose arguments are zero when $\omega = 0$. Therefore, as $\omega \to 0$

(continued from preceding page)

Equation (8): $c_j(t) = \sum_{i=1}^{n} \phi_{ij} f_i(t)$.

Let each forcing function be given by a periodic relationship

$$f_i(t) = f_{o_i}\cos(\ t)$$

where f_{o_i} = amplitude of $f_i(t)$

$\omega = 2\pi/T$ = angular frequency

T = period of forcing function

Then the response can be computed from (9) above in which

$A_{ij}(\omega)$ = an amplitude attenuation factor,

$\theta_{ij}(\omega)$ = a phase shift.

(12) $A_{ij}(\omega)\cos[\omega t + \theta_{ij}(0)] \longrightarrow A_{ij}(0)$

Insertion of (12) in (9) gives

(13) $c_j = \sum_i f_i A_{ij}(0)$

where, for clarity, f_i denotes f_{o_i} and c_j denotes the constant value of $c_j(t)$ which is the resultant state in section j caused by the constants $\{f_i\}$.

There are costs associated with all changes made in the real world. In particular, there are costs associated with any particular set of improvements to the estuary, f_i. It is assumed that the cost of f_1, f_2, ..., f_n = cost of f_1 + cost of f_2 + ... + cost of f_n. Let $V_i(f_i)$ denote the cost of f_i; then the net cost of a set of improved inputs is $\sum_i V_i(m_i)$. The manager would like to minimize $\sum_i V_i(m_i)$ while he obtains the necessary improvements in the estuary's quality. Generally, not all of the n sections need improvement; say that in sections 1, 2, ..., M it is necessary to bring about improvements in the amounts c_1, c_2, ..., c_M.

For example, imagine a five section estuary in two of whose sections it is necessary to raise the annual mean DO from five to eight mg./l. and seven to eight mg./l. respectively. Say that there are treatment plants in three of the five sections. At each of the three plants, it is possible to construct and operate various kinds of additional equipment to decrease the B.O.D. which is now in its effluent. The cost of removing the B.O.D., however, is reduced. In the light of these costs, a manager would seek the combination of actions at the three plants which would, at minimum total cost, increase the DO to at least as high as 8 mg./l. in the two sections of interest.

In this example, M=2, and c_1 and c_2 would correspond to the improvements sought:

$$c_1 = 8 - 5 = 3 \text{ mg./l.}$$

$$c_2 = 8 - 7 = 1 \text{ mg./l.}$$

Here, there are only three sections from which improvements can be made; say that these are sections, two, three and four (the sections need not be numbered consecutively; here, for example, the order of the five sections may be five, four, two, three, one). Therefore, $f_1 = f_5 = 0$. The problem, now, is to find f_2, f_3 and f_4 so that $V_2(f_2) + V_3(f_3) + V_4(f_4)$ is a minimum while both

$$A_{21}(0)f_2 + A_{31}(0)f_3 + A_{41}(0)f_4 \geq 3$$

$$A_{22}(0)f_2 + A_{32}(0)f_3 + A_{42}(0)f_4 \geq 1.$$

Furthermore, it is reasonable to restrict $f_2 \geq 0$, $f_3 \geq 0$ and $f_4 \geq 0$ because inexpensively attained improvement in sections one and two are not sought at the expense of worse quality in the other sections. In other words, improvements in input quality (not total quality) are traded off between sections until the minimum total cost is found which maintains $c_1 \geq 3$ and $c_2 \geq 1$.

The example above may be formulated more generally as a search for values of $f_1, f_2, \ldots, f_n$ which will

$$\text{minimize } \sum_I V_i(f_i)$$

subject to

$$\begin{aligned}
&A_{11}(0)f_1 + A_{21}(0)f_2 + \ldots + A_{n1}(0)f_n \geq c_1 \\
&A_{12}(0)f_1 + A_{22}(0)f_2 + \ldots + A_{n2}(0)f_n \geq c_2 \\
&\vdots \qquad\qquad \vdots \qquad\qquad \vdots \qquad \vdots \\
&A_{1M}(0)f_1 + A_{2M}(0)f_2 + \ldots + A_{nM}(0)f_n \geq c_M
\end{aligned} \tag{14}$$

and $f_1 \geq 0, f_2 \geq 0, \ldots\ldots, f_n \geq 0.$

It may be possible to solve such problems as the one above by visual inspection if the cost functions are linear, there are only three or four unknowns and only two or three inequalities. If, as is more likely for existing estuaries, there are more alternatives and constraints and the cost functions are non-linear (approximable by piecewise linear functions), then the problem may become enormously complicated. Fortunately, considerable attention has been paid by mathematicians to the development of efficient algorithms to solve this kind of problem [see footnote 6]. Note that its solution requires knowledge of $\{V_i(m_i)\}$, the cost functions of improving the inputs into the various sections. Only occasional analysis, however, has been directed at their specifications.[7]

Two initial drawbacks to this formulation are (1) that respectable computational efforts are necessary to solve linear programming problems and (2) the marginal costs have to be determined. Nevertheless, for most problems of interest, computers can do the computation and the data exist from which the costs can be constructed.

The static version of the investment problem could be solved repeatedly for several different sets of incremental improvements in quality (or else a parametric linear programming problem could be formulated: see chapter 8 in the reference of footnote 6). Then the changes in optimal total investment could be related to the resultant incremental increases in quality. These relationships would constitute the marginal cost functions of quality improvement at the various locations which are of interest.

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[7]Logan, J. A., W. D. Hatfield, G. S. Russel and W. R. Lynn, "An Analysis of the Economics of Wastewater Treatment," J. of W.P.C.F., vol. 34, No. 9, pp. 860-992.